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Notes on the Quantization of the Complex Linear Superfield

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ABSTRACT

The quantization of the complex linear superfield requires an infinite tower of ghosts. Using the Batalin-Vilkovisky technique, Grisaru, Van Proeyen, and Zanon have been able to define a correct procedure to construct a gauge-fixed action. We generalize their technique by introducing the Lagrange multipliers into the non-minimal sector and we study the characteristic BRST cohomology. We show how the physical subspace is singled out. Finally, we quantize the model in the presence of a background and of a quantum gauge superfield.

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1 Introduction

The correct quantization of systems with an infinitely reducible gauge symmetry is a longstanding problem. Recently, a new technique for such models has been developed. The authors of [1] showed that a correct implementation of the BV-BRST [2, 3] formalism provides a meaningful quantization procedure for the Complex Linear Superfield.

The complex linear superfield is a constrained field and provides a dual description of the chiral superfield with a different set of auxiliary fields [4, 5, 6, 7]. Solving the constraints in terms of an unconstrained spinor superfield, the redundant unphysical degrees of freedom are removed by new gauge symmetries. The latter, however, turn out to be reducible: there exist non-trivial zero modes of the gauge transformations. This implies that ghosts for ghosts should be introduced in order to have the correct number of degrees of freedom. Furthermore, unlike the case of the antisymmetric tensor field [8], the transformations are infinitely reducible. This means that an infinite tower of ghost fields is indeed required in order to quantize the model covariantly.

In ref. [1], it was shown that, by a simple diagonalization technique, the propagator of the fields can be constructed algorithmically provided a well defined way of fixing the gauge for the tower of ghosts is given. Essentially, the technique is based on a sequence of canonical transformations designed to disentangle the different generations of ghost fields. The sequence of transformation is constructed in such a way that the gauge fixing of the n^{th} order can be done without affecting the $(n-1)^{\text{th}}$ order. However, those diagonalizations, performed by non-local redefinitions, might cause severe problems. We will show that the theory is local and the BRST cohomology changes drastically as a consequence of those transformations.

The case of the linear superfield discussed in refs. [1, 9] is fairly simple. The ungauged version of that model possesses a tower of free ghost fields which are zero modes of the *off-shell* gauge transformations. Therefore, as shown in [1], the BV technique can be used for a correct quantization procedure. Since the ghost fields are free, one can easily compute the functional integral and the correlation functions. The gauged version, on the other hand, is a model with a reducible gauge symmetry where the ghost fields interact with Yang-Mills gauge superfield through supercovariant derivatives. Some years ago, P. Townsend proposed a way to handle such a situation [10]: he suggested to introduce redefined ghost fields such that, in terms of these new variables, the ghosts for ghosts do not interact with the gauge fields. Therefore, by choosing a suitable gauge fixing, the procedure proposed in [1] can be used again to quantize the model. This is one of the aims of the present paper. We also extend Townsend's idea to the case of quantum gauge fields coupled to superfields.

Our interest in the quantization of the linear superfield is mainly due to the extension of the technique developed in [1, 9] to other models with an infinitely reducible gauge symmetry, as the Casalbuoni-Brink-Schwarz superparticle [11, 12, 13, 14, 15], the Green-Schwarz superstring [16, 17, 18, 19, 20, 21], and string field theory [22].

The cases of the Casalbuoni-Brink-Schwarz superparticle and of the Green-Schwarz superstring have some common features with the gauged linear superfield model. In particular, in all these cases the ghosts of the κ-symmetry [12] are interacting with the *physical* fields. Furthermore, the κ-transformations are reducible only *on-shell*. This requires a slightly different quantization technique which has been discussed in [23]. It has been shown there that, by a suitable canonical transformation, the tower of ghost for the κ-symmetry coincides with an off-shell tower of ghosts which describe the same degrees of freedom in suitable variables. Furthermore, the Lorentz gauge-fixing type proposed in [1, 9] cannot be used for the superparticle or the superstring. Instead, the Landau gauge fixing proposed in [13] can be implemented by using an extended non-minimal sector.

In the present paper, we provide a small generalization of the technique of [1] which takes into account the Lagrange multipliers. We also identify the correct number of degrees of freedom and, consequently, the correct physical subspace with the BRST cohomology (or, equivalently, with the antibracket cohomology). We compute the cohomology for the gauge unfixed theory and for the gauge fixed one. In addition, we show that the naive application of the technique of [1] does not provide the correct physical subspace, which can be recovered nevertheless by a careful handling of the non-local canonical transformations.

Here, we provide a complete discussion of the realization of Townsend's idea. In section (2) we review the classical theory and we give a brief account on the BV formalism. For convenience, we follow the notation and the conventions of [1]. In section (3) we show that, by introducing suitable variables for the ghost fields, the technique of [1] can be used to quantize the model. Furthermore, we discuss the antibracket cohomology and the BRST cohomology, and we show that the quantized theory describes the correct degrees of freedom. In particular, we adopt a simpler model (Section (3.2)) to show that the procedure outlined in the previous section can be implemented to all levels of ghosts. In section (4) we discuss the gauged version of the model. Section (5) summarizes our conclusions. The Appendices contain additional technical developments.

2 The classical theory and BV formalism

2.1 The classical action and its gauge symmetry

The kinetic action for a complex linear superfield Σ, $\bar{\Sigma}$, with $\bar{D}^2\Sigma = D^2\bar{\Sigma} = 0$ is

$$S = -\int d^4x d^4\theta \ \bar{\Sigma}\Sigma \ . \tag{2.1}$$

The equivalence of the descriptions of the scalar multiplet by the linear superfield Σ and by the chiral superfield Φ can be exhibited by means of a duality transformation, starting with the action

$$S_D = -\int d^4x d^4\theta \ [\bar{\Sigma}\Sigma + \Phi\Sigma + \bar{\Phi}\bar{\Sigma}] \tag{2.2}$$

(with *unconstrained* Σ and chiral Φ) [4]. Using the equations of motion to eliminate the superfields $\Sigma, \bar{\Sigma}$, leads to the usual chiral superfield action. Eliminating instead the superfields Φ, $\bar{\Phi}$ (whose equations of motion impose the linearity constraint $\bar{D}^2\Sigma = D^2\bar{\Sigma} = 0$) leads to the linear superfield action.

The linearity constraint can be solved in terms of an unconstrained spinor superfield and its complex conjugate by

$$\Sigma = \bar{D}_{\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}} \quad , \quad \bar{\Sigma} = D_\alpha\sigma^\alpha \ , \tag{2.3}$$

and the action becomes

$$S_{cl} = -\int d^4x d^4\theta \ \ D_\alpha\sigma^\alpha \cdot \bar{D}_{\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}} = -\int d^4x d^4\theta \ \sigma^\alpha D_\alpha \bar{D}_{\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}} \ . \tag{2.4}$$

The solution of the constraint has introduced some gauge invariance, since the general spinor superfield σ^α has more components than Σ. Thus the operator $D_\alpha\bar{D}_{\dot{\alpha}}$ is not invertible. We view Σ as the field strength of the gauge field σ^α.

The above action is invariant under the variation $\delta\sigma^\alpha = D_\beta\sigma^{(\alpha\beta)}$ with unconstrained *symmetric* (as indicated by the brackets) bispinor gauge parameter, since $D_\alpha D_\beta$ is antisymmetric in the indices. However, this variation has zero modes, $\delta\sigma^{(\alpha\beta)} = D_\gamma\sigma^{(\alpha\beta\gamma)}$ with the new parameter symmetric in its indices. Similarly, we find zero modes $\delta\sigma^{(\alpha\beta\gamma)} = D_\delta\sigma^{(\alpha\beta\gamma\delta)}$, and so on. Proceeding in this manner one discovers an infinite chain of transformations with zero modes which, upon quantization, leads to an infinite tower of ghosts. This comes about because at every step the ghosts have more components than are necessary to remove gauge degrees of freedom, an apparently unavoidable situation if one wants to maintain manifest Lorentz invariance. It is this feature which makes the quantization of the complex linear superfield difficult.

2.2 Symmetries and the anti-bracket formalism

The essential ingredients in the Batalin-Vilkovisky (BV) [2, 24, 25, 26, 27] formalism are the anti-fields and anti-brackets. For any field ϕ^A, one introduces an anti-field ϕ^*_A. They have statistics opposite to their corresponding fields and a ghost number, assigned such that for the classical fields $gh(\Phi) = 0$, the (extended) action has ghost number zero, and for all fields $gh(\phi^*_A) = -gh(\phi^A) - 1$.

The anti-bracket of two functions F and G is defined by

$$(F, G) = \frac{\partial_r}{\partial \phi^A} F \frac{\partial_l}{\partial \phi^*_A} G - \frac{\partial_r}{\partial \phi^*_A} F \frac{\partial_l}{\partial \phi^A} G \,, \tag{2.5}$$

where $\partial_{r/l}$ denote the derivative from the left or from the right. They satisfy graded commutation, distribution and Jacobi relations [2]. For these brackets, fields and anti-fields behave as coordinates and their conjugate momenta

$$(\phi^A, \phi^B) = 0 \ ; \quad (\phi^*_A, \phi^*_B) = 0; \quad (\phi^A, \phi^*_B) = \delta^A_B. \tag{2.6}$$

The extended action S is the solution of the master equation

$$(S, S) = 0 \,, \tag{2.7}$$

with the boundary condition that the classical action S_{class} coincides with minimal action when the ghost fields are set to zero and the anti-fields ϕ^*_A are coupled to the gauge transformations of the classical fields. Notice that there is a natural grading among the fields and the anti-fields, namely the antighost number [25]. This allows a convenient decomposition of the extended action and, accordingly, the master equation can be easily solved.

Canonical transformations are an important part of the formalism [24]. They preserve the anti-bracket structure (2.5): calculating the anti-brackets in the old or new variables is the same, or, in other words, the new variables also satisfy (2.6). Therefore also the master equation $(S, S) = 0$ is preserved under such transformations.

Canonical transformations from $\{\phi \, \phi^*\}$ to $\{\phi' \, \phi'^*\}$, for which the matrix $\left.\frac{\partial_r \phi^B}{\partial \phi'^A}\right|_{\phi'^*}$ is invertible, can be obtained from a fermionic generating function $F(\phi, \phi'^*)$. The transformations are defined by

$$\phi'^A = \frac{\partial F(\phi, \phi'^*)}{\partial \phi'^*_A} \qquad\qquad \phi^*_A = \frac{\partial F(\phi, \phi'^*)}{\partial \phi^A}. \tag{2.8}$$

- Point transformations are the easiest ones. These are just redefinitions of the fields $\phi'^A = f^A(\phi)$. Their generating function is $F = \phi'^*_A f^A(\phi)$ which thus determines the corresponding transformations of the anti-fields. The latter replace the calculations of the variations of the new variables.

- Adding the BRST transformation of a function, $s\Psi(\phi)$, to the action is obtained by a canonical transformation with $F = \phi'^*_A \phi^A + \Psi(\phi)$. The latter gives

$$\phi'^A = \phi^A \; ; \qquad \phi^*_A = \phi'^*_A + \partial_A \Psi(\phi). \tag{2.9}$$

  and by means of these transformations it is possible to implement the gauge fixing conditions as a canonical change of variables.

The canonical transformations leave by definition the master equation invariant, and because they are non-singular, they also keep the properness requirement on the extended action. We will discuss this point at length later on. Of course in the new variables, we do not see the classical limit anymore. But the most important property is that the anti-bracket cohomology [27] (and the BRST cohomology) is not changed.

This minimal solution, or the so-called extended action, coincides with the classical action when the anti-fields are set to zero. The anti-field dependent terms encode the structure of the symmetry algebra: the terms linear in the anti-fields generate the quantum BRST transformation of the fields, according to the formula

$$\gamma \Phi^A = (\Phi^A, S)|_{\phi^*=0} \tag{2.10}$$

while the higher-order terms contain the non-closure functions of the algebra. γ defines a coboundary operator and the space of physical degrees of freedom is identified with the cohomology group $H^*(\gamma)$ with zero ghost number. This defines the so-called BRST characteristic cohomology [26]. Alternatively and equivalently, one can compute the antibracket cohomology singled out by means of the differential

$$s\Phi^A = (\Phi^A, S)\,, \qquad s\Phi^*_A = (\Phi^*_A, S)\,. \tag{2.11}$$

In the case of the antibracket cohomology, the physical subspace is selected by decomposing the differential $s = \gamma + \delta$, precisely into the BRST differential γ and the Koszul-Tate resolution δ, and identifying the physical degrees of freedom by the cohomology $H^*(\gamma, H^*(\delta))$. As shown in [27], the two cohomological groups coincide.

2.3 GPZ technique for infinite ghost towers

Following the conventions of [1], the minimal classical action (2.4) containing all the minimal zero modes and the couplings with their corresponding antifields is written as

$$S_{min} = \bar{\sigma}^{\dot{\alpha}} \bar{D}_{\dot{\alpha}} D_\alpha \sigma^\alpha \; + \sum_{i=1}^{\infty} \sigma^*_{A_i} D_\beta \sigma^{(\beta A_i)} + \sum_{i=1}^{\infty} \bar{\sigma}^{(\dot{\beta} \dot{A}_i)} \bar{D}_{\dot{\beta}} \bar{\sigma}^*_{\dot{A}_i} \,, \tag{2.12}$$

where A_i denotes the symmetrized set of indices $(\alpha_1, \ldots, \alpha_2)$.

According to the BV formalism, and indicating the antifields of the antighosts respectively by $b^{*\alpha}_{\dot{\alpha}}$ and $\bar{b}^{*\dot{\alpha}}_{\alpha}$, we add to S_{min} the non–minimal term

$$S_{nm,1} = \bar{b}^{*\dot{\alpha}}_{\alpha} b^{*\alpha}_{\dot{\alpha}} \,, \tag{2.13}$$

and we perform the canonical transformations generated by the gauge fermion

$$\Psi_1 = b^{\dot{\alpha}}_{\alpha} \bar{D}_{\dot{\alpha}} \sigma^\alpha + \bar{\sigma}^{\dot{\alpha}} D_\alpha \bar{b}^{\alpha}_{\dot{\alpha}} \,, \tag{2.14}$$

which shifts b^*, σ^* antifields of the antighost and of σ field, respectively (and the corresponding hermitian counterparts). Thus, the 0^{th}-level quantized action reads

$$\begin{aligned} S_1 &= S_{Q,1} + S_{0,1} + S_{*,1} + \sum_{i=1}^{\infty} \sigma^*_{A_i} D_\beta \sigma^{(\beta A_i)} + \sum_{i=1}^{\infty} \bar{\sigma}^{(\dot{\beta} \dot{A}_i)} \bar{D}_{\dot{\beta}} \bar{\sigma}^*_{\dot{A}_i} \,, \\ S_{Q,1} &= \bar{\sigma}^{\dot{\alpha}} i \partial_{\dot{\alpha}\alpha} \sigma^\alpha \,, \\ S_{0,1} &= b^{\dot{\alpha}}_{\alpha} \bar{D}_{\dot{\alpha}} D_\beta \sigma^{(\beta\alpha)} + \text{h.c.} \,, \\ S_{*,1} &= \bar{b}^{*\dot{\alpha}}_{\alpha} b^{*\alpha}_{\dot{\alpha}} + \bar{b}^{*\dot{\alpha}}_{\alpha} \bar{D}_{\dot{\alpha}} \sigma^\alpha + \bar{\sigma}^{\dot{\alpha}} D_\alpha b^{*\alpha}_{\dot{\alpha}} \,. \end{aligned} \tag{2.15}$$

As explained in [1], in order to remove the couplings between the antifields $b^*, \bar{b}^*$ and the 0^{th}-level quantized fields, it is convenient to redefine the variables by a canonical transformations generated by the fermion

$$F(\Phi, \tilde{\Phi}^*) = \Phi^A \tilde{\Phi}^*_A - \tilde{\bar{b}}^{*\dot{\alpha}}_{\alpha} \bar{D}_{\dot{\alpha}} \frac{1}{\Box} i \partial^{\alpha\dot{\beta}} \tilde{\bar{\sigma}}^*_{\dot{\beta}} - \tilde{\sigma}^*_\beta \frac{1}{\Box} i \partial^{\beta\dot{\alpha}} D_\alpha \tilde{b}^{*\alpha}_{\dot{\alpha}} \;. \tag{2.16}$$

We notice that those transformations are non-local and they affect both the couplings between $b^*, \bar{b}^*$ with σ and $\bar{\sigma}$ and the minimal terms involving $\sigma^*, \bar{\sigma}^*$. These redefinitions might cause severe problems for the locality of the gauge fixed theory. In the forthcoming sections, we analyze carefully the problem and we show how the non-locality is essential to compute the BRST characteristic cohomology.

Finally, in order to fix the gauge for ghosts and for extra ghosts, one has to add non-minimal terms to the action

$$
\begin{aligned}
S_0 &= S_{cl} + \sum_{i=1}^{\infty} \sigma^*_{A_i} D_\beta \sigma^{(\beta A_i)} + \sum_{i=1}^{\infty} \bar{\sigma}^{(\dot{\beta}\dot{A}_i)} \bar{D}_{\dot{\beta}} \bar{\sigma}^*_{\dot{A}_i} + S_{f,L} + S_{f,R} \\
S_{f,L} &= \sigma^*_{\alpha_1\alpha_2} C^{\alpha_1\alpha_2}\lambda + \sigma^*_{A_2\alpha_3} C^{\alpha_2\alpha_3}\lambda^{\alpha_1} \\
&\quad + \sigma^*_{A_3\alpha_4} C^{\alpha_3\alpha_4}\lambda^{A_2} + \varsigma^* C_{\alpha_1\alpha_2}\lambda^{\alpha_1\alpha_2} + \ldots\ ,
\end{aligned} \tag{2.17}
$$

where $S_{f,R}$ is the complex conjugate of $S_{f,L}$. We do not repeat here the considerations of [1] which justified the introduction of those new ghosts and their corresponding antighosts. However, we would like to underline that this is a specific feature of the present model. In the case of the Casalbuoni-Brink-Schwarz superparticle and of the Green-Schwarz superstring, this phenomenon does not occur, since the Lorentz representation of κ-symmetry ghosts does not change from one level to another.

The gauge fixing of ghosts and of other non-minimal fields is obtained in the same way as the 0^{th}-level, i.e. by introducing the non-minimal couplings

$$
\begin{aligned}
S_{nm,2} &= d^{*\dot{\alpha}}_{A_2} b^{*A_2}_{\dot{\alpha}} + \nu^*\mu^* + h.c. \\
S_{nm,3} &= \bar{e}^{*\dot{A}_2}_{A_2} e^{*A_2}_{\dot{A}_2} + \left(d^{*\dot{\alpha}}_{A_3} b^{*A_3}_{\dot{\alpha}} + \nu^*_\alpha \mu^{*\alpha} + d^* b^* + \text{h.c.} \right) \\
&\quad - \bar{\rho}^*_\alpha i\partial^{\alpha\dot{\alpha}} \rho^*_{\dot{\alpha}} - \bar{\rho}'_\alpha i\partial^{\alpha\dot{\alpha}} \rho'_{\dot{\alpha}}\,,
\end{aligned} \tag{2.18}
$$

and by performing the following canonical transformations

$$
\begin{aligned}
\Psi_2 &= b^{\dot{\alpha}}_\alpha D_\beta d^{(\beta\alpha)}_{\dot{\alpha}} + b^{\dot{\alpha}}_\alpha i\partial^\alpha{}_{\dot{\alpha}}\nu + b^{\dot{\alpha}}_{A_2} \bar{D}_{\dot{\alpha}} \sigma^{A_2} + \frac{1}{2}\mu C_{\alpha\beta}\sigma^{\beta\alpha} + \text{h.c.} \\
\Psi_3 &= b^{\dot{\alpha}}_{A_3} \bar{D}_{\dot{\alpha}} \sigma^{A_3} + \frac{2}{3}\mu_{\alpha_1} C_{\alpha_2\alpha_3}\sigma^{A_2\alpha_3} + b^{\dot{\alpha}}_{A_2}\left(D_{\alpha_3} d^{A_3}_{\dot{\alpha}} + i\partial^{\alpha_1}_{\dot{\alpha}}\nu^{\alpha_2} + i\partial^{\alpha_1}_{\dot{\alpha}} D^{\alpha_2} d \right) \\
&\quad + e^{\dot{A}_2}_{A_2} \bar{D}_{\dot{\alpha}_1} d^{A_2}_{\dot{\alpha}_2} - 2\rho^{\dot{\alpha}} \bar{D}_{\dot{\alpha}}\nu + \text{h.c.}\ .
\end{aligned} \tag{2.19}
$$

Again, to remove the mixing between the fields of the 1^{st}-level and those of the 2^{nd}-level, one has to perform suitable diagonalizations, which are similar to the non-local transformations generated by (2.16). The non-minimal terms of (2.18) contain also the Nielsen-Kallosh ghosts [28], necessary in order to compensate the extra-propagating modes as discussed in [1].

In the same spirit as above, one continues to fix all the ghost kinetic terms. Since we are not interested to re-derive the full result of [1], we will refer to that paper also for the complete resulting action. In the forthcoming sections, we will show how the same result can be obtained by introducing suitable Lagrange multipliers associated with the antighost fields and we construct the corresponding canonical transformations to decouple different levels before the gauge-fixings.

3 BRST cohomology and physical degrees of freedom

3.1 First levels

In order to provide a clear description of the cohomology computation for the gauge unfixed and gauge fixed theory, we introduce the Lagrange multipliers associated to the antighost fields. The counting of classical degrees of freedom does not change, but the computation of the gauge-fixed degrees of freedom is simplified. Furthermore, as explained in the introduction, for some models a suitable gauge fixing can be only implemented by means of the Lagrange multipliers (cf. [23]).

We introduce the Lagrange multipliers $\beta^{\alpha}_{\dot\alpha}, \delta^{A_2}_{\dot\alpha}, \beta^{\dot\alpha}_{A_2}, \dots$ associated with the antighost fields and with the extra ghost fields $b^{\alpha}_{\dot\alpha}, d^{A_2}_{\dot\alpha}, b^{\dot\alpha}_{A_2}, \dots$ displayed in table (1). [1] The Lagrange multipliers obey:

$$
\begin{array}{llllll}
s\, b^{\alpha}_{\dot\alpha} = \beta^{\alpha}_{\dot\alpha}\,, & s\, d^{A_2}_{\dot\alpha} = \delta^{A_2}_{\dot\alpha}\,, & s\,\nu = n\,, & s\, b^{\dot\alpha}_{\dot A_2} = \beta^{\dot\alpha}_{\dot A_2}\,, & s\,\mu = m\,, & \dots\,, \\
s\,\beta^{\alpha}_{\dot\alpha} = 0\,, & s\,\delta^{A_2}_{\dot\alpha} = 0\,, & s\, n = 0, & s\,\beta^{\dot\alpha}_{\dot A_2} = 0\,, & s\, m = 0\,, & \dots\,,
\end{array} \tag{3.1}
$$

where s is the generators of the classical BRST transformations. As a convention, we denote with a Greek letter the Lagrange multiplier if the corresponding ghost fields is denoted by a Latin letter, and vice-versa. In table (2) the Lagrange multipliers are shown, together with their spinorial indices and their quantum numbers.

Using the Lagrange multiplier, in place of (2.13), we add the non-minimal terms

$$
S_{nm,1} = s\left(\bar b^{*\dot\alpha}_{\alpha} b^{\alpha}_{\dot\alpha} + \bar b^{\dot\alpha}_{\alpha} b^{*\alpha}_{\dot\alpha}\right) = \bar b^{*\dot\alpha}_{\alpha}\beta^{\alpha}_{\dot\alpha} + \bar\beta^{\dot\alpha}_{\alpha} b^{*\alpha}_{\dot\alpha}\,, \tag{3.2}
$$

and we generate the canonical transformations on the fields of the minimal action S_{min} of eq. (2.17) by means of the gauge fermion

$$
\Psi_1 = b^{\dot\alpha}_{\alpha}\left(\bar D_{\dot\alpha}\sigma^{\alpha} + k\beta^{\alpha}_{\dot\alpha}\right) + \left(\bar\sigma^{\dot\alpha} D_{\alpha} + \bar k\bar\beta^{\dot\alpha}_{\alpha}\right)\bar b^{\alpha}_{\dot\alpha}\,. \tag{3.3}
$$

Here, k is a complex gauge parameter (cf. [1] for a discussion on gauge parameters in the quantization of the complex linear superfield). The resulting action is given by

$$
\begin{aligned}
S'_1 &= S'_{Q,1} + S_{0,1} + S'_{*,1} + \sum_{i=1}^{\infty}\sigma^{*}_{A_i} D_{\beta}\sigma^{(\beta A_i)} + \sum_{i=1}^{\infty}\bar\sigma^{(\dot\beta\dot A_i)}\bar D_{\dot\beta}\bar\sigma^{*}_{\dot A_i}\,, \\
S'_{Q,1} &= \bar\sigma^{\dot\alpha}\bar D_{\dot\alpha} D_{\alpha}\sigma^{\alpha} + \bar\beta^{\dot\alpha}_{\alpha}\bar D_{\dot\alpha}\sigma^{\alpha} + \bar\sigma^{\dot\alpha} D_{\alpha}\beta^{\alpha}_{\dot\alpha} + \left(\bar k + k\right)\bar\beta^{\dot\alpha}_{\alpha}\beta^{\alpha}_{\dot\alpha} \qquad (3.4) \\
S_{0,1} &= b^{\dot\alpha}_{\alpha}\bar D_{\dot\alpha} D_{\beta}\sigma^{(\beta\alpha)} + \bar\sigma^{(\dot\beta\dot\alpha)\bar D_{\dot\beta}} D_{\alpha}\bar b^{\alpha}_{\dot\alpha}\,, \\
S'_{*,1} &= \bar b^{*\dot\alpha}_{\alpha}\beta^{\alpha}_{\dot\alpha} + \bar\beta^{\dot\alpha}_{\alpha} b^{*\alpha}_{\dot\alpha}\,. \qquad (3.5)
\end{aligned}
$$

[1]Following [1] the symbol $\sigma^{\alpha\beta}$ contains both the symmetric part (the minimal ghost) and the anti-symmetric part (the non-minimal ghost): $\sigma^{\alpha\beta} = \sigma^{(\alpha\beta)} + C^{[\alpha\beta]}\lambda$.

The kinetic terms for the superfield σ^α and the Lagrange multiplier $\beta^\alpha_{\dot\alpha}$ (and their hermitian partners) provide a good propagator which can be easily computed. The formulation with the Lagrange multiplier is clearly equivalent to the formulation of [1]. This can be seen by eliminating the Lagrange multipliers by means of their equations of motion, which are

$$-\beta^\alpha_{\dot\alpha} + \bar D_{\dot\alpha}\sigma^\alpha + b^{*\alpha}_{\dot\alpha} = 0, \qquad \text{for} \;\; k+\bar k = 1, \tag{3.6}$$

and its hermitian conjugate.

Before discussing the canonical transformations (2.16), we note that, due to the trivial BRST variations of the antighosts and the Lagrange multipliers described in eqs. (3.1), it is easy to eliminate those fields from the BRST cohomology $H^*(\gamma)$ for zero ghost number. On the other hand, the BRST transformations of the spinor fields σ^α do remove the unphysical degrees of freedom. At this level, the BRST charge is easily constructed and the physical subspace does coincide with the classical one.

Following the suggestions of [1], we perform the diagonalization in order to decouple the fields of the 0^{th}-level from the ghost fields in such a way that the further gauge fixings do not modify the structure of the 0^{th}-level field action. The necessary canonical transformations involve also the Lagrange multipliers:

$$\sigma^\alpha \longrightarrow \sigma^\alpha + F(\partial)^{\alpha\dot\tau}_{\tau} b^{*\tau}_{\dot\tau} \qquad \beta^\alpha_{\dot\alpha} \longrightarrow \beta^\alpha_{\dot\alpha} + G(\partial)^{\alpha\dot\tau}_{\dot\alpha\tau} b^{*\tau}_{\dot\tau}\,, \tag{3.7}$$

and equivalently for the hermitian conjugates. In the above equation, $F(\partial)^{\dot\tau\alpha}_{\tau}$ and $G(\partial)^{\dot\tau\alpha}_{\tau\dot\alpha}$ are integro-differential operators fixed by cancelling the couplings $S'_{*,1}$ in eq. (3.5). Notice that as a consequence of the canonical transformations (3.7), the antighosts $b^\alpha_{\dot\alpha}$ and $\bar b^{\dot\alpha}_\alpha$ are also modified by

$$b^{\dot\alpha}_\alpha \longrightarrow b^{\dot\alpha}_\alpha + \sigma^*_\tau F(\partial)^{\dot\alpha\tau}_\alpha + \beta^{*\dot\tau}_\tau G(\partial)^{\dot\alpha\tau}_{\alpha\dot\tau}\,, \tag{3.8}$$

where the operators $F(\partial)^{\dot\alpha\tau}_\alpha, G(\partial)^{\dot\alpha\tau}_{\alpha\dot\tau}$ act on the right. After inserting the transformed variables in action (3.5), the couplings of the antifields of the $b^\alpha_{\dot\alpha}$ with 0^{th}-level fields and with themselves are

$$\begin{aligned} S'_{Q,1} + S'_{*,1} \longrightarrow \qquad & \bar\sigma^{\dot\alpha}\left(\bar D D F + D G\right)^{\alpha\dot\tau}_{\tau} b^{*\tau}_{\dot\tau} + \\ & \bar\beta^\alpha_{\dot\alpha}\left(\bar D F + (k+\bar k) G + 1\right)^{\alpha\dot\tau}_{\dot\alpha\,\tau} b^{*\tau}_{\dot\tau} + \text{h.c.} + \\ & \bar b^{*\dot\alpha}_\alpha\left(\bar F \bar D D F + \bar F D G + \bar G \bar D F + (k+\bar k)\bar G G + \bar G + G\right)^{\dot\alpha\,\tau}_{\alpha\dot\tau} b^{*\tau}_{\dot\tau}\,. \end{aligned} \tag{3.9}$$

In order to avoid a cumbersome notation by making all the spinor indices explicit, we use the superfield matrix notation of [4]. The operators $\bar F$ and $\bar G$ are the hermitian conjugates of F, G plus suitable integrations by parts. As a consequence of the canonical transformation, we obtain also the following terms

$$\begin{aligned} \sigma^*_\alpha D_\beta \sigma^{(\alpha\beta)} + b^{\dot\alpha}_\alpha \bar D_{\dot\alpha} D_\beta \sigma^{(\beta\alpha)} + \text{h.c.} \longrightarrow \quad & \sigma^*_\alpha \left(1 + F\bar D\right)^\alpha_{\ \tau} D_\beta \sigma^{(\tau\beta)} + \\ & \beta^{*\dot\alpha}_\alpha \left(G\bar D\right)^\alpha_{\ \dot\alpha\tau} D_\beta \sigma^{(\tau\beta)} + \text{h.c.}\,. \end{aligned} \tag{3.10}$$

Using the solution given in [1], and the relation $1 - \Box^{-1}\Box = 1 - \Box\Box^{-1} = K_0$, where K_0 is the projector on the zero modes of the d'Alembertian $\Box$, we have

$$F(\partial)^{\dot\tau\alpha}_{\tau} = -\frac{1}{\Box} i\partial^{\dot\tau\alpha} D_\tau\,, \qquad G(\partial)^{\dot\tau\alpha}_{\tau\dot\alpha} = -\frac{1}{k+\bar k}\left(1 - \frac{i\,\partial\!\!\!/}{\Box}\bar D D\right)^{\dot\tau\alpha}_{\tau\dot\alpha}\,. \tag{3.11}$$

Notice that, treating the integro-differential operators $F(\partial)$ and $G(\partial)$ formally, one misses the contribution of zero modes. Indeed, only a carefully handling of the definition of the inverse of $\Box$ introduces the operator K_0. We do not need to specify K_0, but we want to underline the fact that it is incorrect to neglect such contribution.

From eq. (3.9) and setting $k = \bar k = -1/2$, we have

$$S'_{Q,1} + S'_{*,1} \longrightarrow \bar\sigma^{\dot\alpha} K_0 D_\tau b^{*\tau}_{\dot\alpha} + \text{h.c.} + \bar b^{*\dot\alpha}_\alpha \left(1 + \frac{i\,\partial\!\!\!/}{\Box}\bar D D(1-K_0)\right)^{\dot\alpha\,\tau}_{\alpha\dot\tau} b^{*\tau}_{\dot\tau}\,. \tag{3.12}$$

From eq. (3.10):

$$\begin{aligned} \sigma^*_\alpha D_\beta \sigma^{(\alpha\beta)} + b^{\dot\alpha}_\alpha \bar D_{\dot\alpha} D_\beta \sigma^{(\beta\alpha)} + \text{h.c.} \longrightarrow \quad & \sigma^*_\alpha K_0 D_\beta \sigma^{(\alpha\beta)} + \\ & \beta^{*\dot\alpha}_\alpha K_0 \bar D_{\dot\alpha} D_\beta \sigma^{(\alpha\beta)} + \text{h.c.}\,. \end{aligned} \tag{3.13}$$

We must notice that the integro-differential operators $F(\partial)$ and $G(\partial)$ cannot cancel all the possible contributions found in [1]: the contribution of zero modes is left. For the purpose of the definition of the propagator, this detail is inessential, but it plays a fundamental role in the cohomological computation. Indeed, we will show that the further gauge fixing does not affect the 0^{th}-level fields (as expected on the basis of the GPZ technique).

Applying the definition (2.10), we have

$$\begin{aligned} \gamma\,\sigma^\alpha &= K_0 D_\beta \sigma^{(\alpha\beta)}\,, \\ \gamma\, b^\alpha_{\dot\alpha} &= K_0 \bar D_{\dot\alpha} \sigma^\alpha\,, \\ \gamma\,\beta^\alpha_{\dot\alpha} &= K_0 \bar D_{\dot\alpha} D_\beta \sigma^{(\alpha\beta)}\,, \end{aligned} \tag{3.14}$$

where the only surviving contributions are the zero modes. This ensures the nilpotency of the BRST differential γ together with the equations of motion

$$\bar D_{\dot\alpha} D_\beta \sigma^\beta + D_\alpha \beta^\alpha_{\dot\alpha} = 0\,, \qquad \bar D_{\dot\alpha}\sigma^\alpha - \beta^\alpha_{\dot\alpha} = 0\,. \tag{3.15}$$

By means of the equations of motion, it is immediate to see that the degrees of freedom of the Lagrange multiplier depend on σ^α. Finally, the non-local BRST transformations are enough to show that, among the propagating degrees of freedom, only the physical ones belong to the cohomology $H^*(\gamma)$. This selects the correct physical subspace. Notice the non-locality of the BRST transformations induced by the canonical transformations (3.7) and (3.8).

This concludes the analysis of the first level. The other levels can be studied in the same way. Indeed, one needs the quantized version of the ghost fields in order to extend the constraints of the gauge fixing to all orders.

Along the lines of the above procedure, one can easily (although tediously) construct the all-level quantization of the Complex Linear Superfield. However, already at the second level, cumbersome expressions hide completely the structure of cohomology and the computation of the physical spectrum. For that reason we study here a simpler model, for which the complete (all-levels) action can be easily computed.

3.2 A toy model

The main points of the technique we are using to study infinitely reducible systems and their cohomology $H(\gamma, H(\delta))$ can be elucidated on a simple model, which resembles in many respects the Casalbuoni-Brink-Schwarz [11, 12, 13, 14, 15] superparticle, or, more exactly, its fermionic sector. In particular, the structure of the symmetry and the field content of the two models are the same, while the details of the interactions between the various fields account for the non trivial differences.

The model is a $0+1$-dimensional theory of a $SO(9,1)$ Majorana-Weyl spinor θ (the ten-dimensional Lorentz group is here simply an internal symmetry, as it is the case for the world-volume theories of supermembranes), with the classical action

$$S_{cl} = -\bar{\theta}\sigma_{-}\partial\theta \,. \tag{3.16}$$

It is immediate to see that this model has a gauge symmetry

$$\delta\theta = \sigma_{-}\kappa_1 \,,$$

with an infinite chain of zero modes:

$$\begin{aligned}
\delta\kappa_1 &= \sigma_{-}\kappa_2 \,, \\
&\vdots \\
\delta\kappa_n &= \sigma_{-}\kappa_{n+1} \,, \\
&\vdots
\end{aligned}$$

The gauge symmetry cannot be used to eliminate the unphysical degrees of freedom of θ without breaking the $SO(9,1)$ covariance, so we have to quantize the infinitely reducible theory. The minimal action can be written at once:

$$\begin{aligned} S_{min} &= S_{cl} + S_* \, , \\ S_* &= \bar{\theta}^* \sigma_- k_1 + \sum_{p \geq 1} \bar{k}_p^* \sigma_- k_{p+1} \, . \end{aligned}$$

The gauge-fixing procedure starts with the introduction of non-minimal terms

$$S_{nm,1} = \bar{\chi}_1^{1*} \pi_1^1$$

followed by a canonical transformation generated by

$$\Psi_1 = \bar{\chi}_1^1 \partial \theta$$

which yields

$$S_1' = S_{min} + (\bar{\chi}_1^{1*} - \bar{\theta}\partial)\pi_1^1 - \bar{\chi}_1^1 \sigma_- \partial k_1 \, .$$

We see that the coupling of θ and π_1^1 provides a well-defined propagator for these fields, so that the gauge fixing in this sector is accomplished, but we have also generated a Lagrangian for the first ghost k_1 which, in turn, needs a gauge fixing. We want to perform this gauge fixing without affecting the sector already fixed. This can be done if we are able to decouple this sector from the ghosts. In particular, since the antifield of χ_1^1 will generate other terms which will interact with π_1^1, we look for a redefinition of the fields that cancels this coupling. It turns out that we can not completely cancel it: the canonical transformation we need for the diagonalization is generated by

$$\Xi_1 = \bar{\theta}^* \partial^{-1} \chi_1^{1*} \, .$$

It corresponds to a redefinition of θ and χ_1^1. The operator ∂^{-1} is the formal inverse of the derivative. However, since the derivative has a kernel, given by the constant functions, it can only be inverted on the subspace orthogonal to its kernel, so that the following relation holds:

$$\partial \partial^{-1} = \partial^{-1} \partial = 1 - P_0$$

where P_0 is the projector on the kernel. This has the consequence that the projected part of the coupling survives; this is enough for our purposes, since the shifts of the fields due to the gauge-fixings are always in the form of derivatives, so that they are annihilated by the projector. The action after the diagonalization is

$$\begin{aligned} S_1'' &= -\bar{\theta}\sigma_- \partial \theta - \bar{\theta}\partial \pi_1^1 - \bar{\chi}_1^1 \sigma_- \partial k_1 \\ &+ \bar{\theta}^* P_0 \sigma_- k_1 + S_{*,2} \\ &+ \bar{\chi}_1^{1*} P_0 \pi_1^1 - \bar{\chi}_1^{1*}(1 + P_0)\sigma_- \partial^{-1} \chi_1^{1*} \, . \end{aligned}$$

It is worthwhile to analyze the BRST characteristic cohomology now. The equations of motion set all the fields to constant values, or, in other words, only the projected component under P_0 is non-vanishing on shell. The BRST transformations are

$$\begin{aligned}
\gamma\theta &= P_0\sigma_- k_1\,,\\
\gamma\pi_1^1 &= 0\,,\\
\gamma k_n &= \sigma_- k_{n+1}\,,\\
\gamma\chi_1^1 &= P_0\pi_1^1\,.
\end{aligned}$$

The last transformation shows that $(P_0\chi_1^1, P_0\pi_1^1)$ form a trivial pair. If we missed the contribution of the zero modes, we would not find at this point the correct physical spectrum, since the constant part of π_1^1 would be part of the cohomology. It should be noted also that the diagonalization generates non-local terms in the antifields (the last term in eq. (3.17)). These could lead, after successive gauge-fixings, to non-local terms in the Lagrangian. This, however, does not happen, as all the potentially dangerous contributions can be shown to vanish.
The procedure can now be continued, fixing the gauge at any level and then performing the diagonalization before moving to the next level. Since our interest does not lie in this particular model, we do not present the details of the successive diagonalizations, but just point out that in this case it is not difficult to complete the procedure to all orders and give the complete action and BRST charge. The result is as follows:

$$\begin{aligned}
S &= -\bar\theta\sigma_-\partial\theta - \bar\theta\partial\pi_1^1 - \sum_{n\geq 1}\sum_{p=0}^{m}\bar\chi_n^p\partial\pi_{n+1}^{p+1} - \sum_{n\geq 1}\sum_{p=0}^{[n/2]}\bar\chi_n^p\sigma_-\partial\chi_n^{p-1} - \sum_{n\geq 1}\bar\chi_{2n}^{2n}\sigma_-\partial\chi_{2n}^{2n}\\
&+ \sum_{n\geq 0}\bar k_n^*\sigma_- P_0\left(k_{n+1} + \partial^{-1}\chi_{n+2}^{1*}\right) + \sum_{n\geq 1}\sum_{p=1}^{n}\bar\chi_n^{p*}P_0\pi_n^p\\
&- \sum_{n\geq 2}\sum_{p=1}^{[n/2]}\bar\chi_n^{2p*}\sigma_-(1+P_0)\partial^{-1}\chi_n^{2p-1*} - \sum_{n\geq 0}\bar\chi_{2n+1}^{2n+1*}\sigma_-(1+P_0)\partial^{-1}\chi_{2n+1}^{2n+1*}\,. \qquad (3.17)
\end{aligned}$$

4 Quantization of the gauged Complex Linear Superfield

The quantization of the linear superfield coupled to gauge fields has been discussed in refs. [1, 9]. The methods of these papers are reviewed in the next section; here, we first review the idea of P. Townsend presented in the lectures [10] and we show how this idea can be implemented in the case of linear superfield coupled to gauge field.

The coupling of Σ to the Yang-Mills superfield V is easily accomplished by defining covariant derivatives and covariantly linear superfields with respect to the gauge fields. We use the conventions in ref. [4]. Thus we consider $N = 1$ superfields V, Σ and $\bar{\Sigma}$ Lie-algebra valued in the adjoint representation, $V = V^a T_a$, $\Sigma = \Sigma^a T_a$, $\bar{\Sigma} = \bar{\Sigma}^a T_a$ with $[T_a, T_b] = i f^c{}_{ab} T_c$ and $tr T_a T_b = K \delta_{ab}$, and we introduce covariant derivatives, in vector representation (see [4] and Appendix B),

$$\nabla_\alpha = e^{-W} D_\alpha e^W \,, \qquad \bar{\nabla}_{\dot{\alpha}} = e^{\bar{W}} \bar{D}_{\dot{\alpha}} e^{-\bar{W}} \,, \qquad \nabla_{\alpha\dot{\alpha}} = -i\{\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}\} \,.$$

According to [1, 9], in the classical action (suppressing the superspace integrals) , the superderivatives are replaced by the covariant superderivatives:

$$S_{\rm cl} = {\rm tr}\left(\bar{\sigma}^{\dot{\alpha}} \bar{D}_{\dot{\alpha}} D_\alpha \sigma^\alpha\right) \longrightarrow {\rm tr}\left(\bar{\sigma}^{\dot{\alpha}} \bar{\nabla}_{\dot{\alpha}} \nabla_\alpha \sigma^\alpha\right) = {\rm tr}\left(\bar{\sigma}^{\dot{\alpha}} e^{\bar{W}} \bar{D}_{\dot{\alpha}} e^{-V} D_\alpha e^W \sigma^\alpha\right) \,. \tag{4.1}$$

The trace is computed over the representation of matter fields σ, and $e^V = e^W e^{\bar{W}}$ defines the real superfield V in terms of its chiral counterparts. Introducing the redefined matter fields $\hat{\sigma} = e^W \sigma$ and $\hat{\bar{\sigma}} = \bar{\sigma}\, e^{\bar{W}}$, we can rewrite the classical action in the form

$$S_{\rm cl} = {\rm tr}\left(\hat{\bar{\sigma}}^{\dot{\alpha}} \bar{D}_{\dot{\alpha}} D_\alpha \hat{\sigma}^\alpha\right) + {\rm tr}\left(\hat{\bar{\sigma}}^{\dot{\alpha}} \bar{D}_{\dot{\alpha}} \left(e^{-V} - 1\right) D_\alpha \hat{\sigma}^\alpha\right) \,. \tag{4.2}$$

The second term is an interaction term starting at order $O(V)$ in the real superfield. In terms of the redefined fields, the new minimal action is

$$\begin{aligned} S_{min} &= S_{\rm cl} + \sum_{i=1}^{\infty} \hat{\sigma}^*_{A_i} D_\beta \hat{\sigma}^{(\beta A_i)} + \sum_{i=1}^{\infty} \hat{\sigma}^{(\dot{\beta}\dot{A}_i)} \bar{D}_{\dot{\beta}} \hat{\sigma}^*_{\dot{A}_i} + S_{f,L} + S_{f,R} \\ S_{f,L} &= \hat{\sigma}^*_{\alpha_1\alpha_2} C^{\alpha_1\alpha_2} \hat{\lambda} + \hat{\sigma}^*_{A_2\alpha_3} C^{\alpha_2\alpha_3} \hat{\lambda}^{\alpha_1} \\ &\quad + \hat{\sigma}^*_{A_3\alpha_4} C^{\alpha_3\alpha_4} \hat{\lambda}^{A_2} + \hat{\varsigma}^* C_{\alpha_1\alpha_2} \hat{\lambda}^{\alpha_1\alpha_2} + \dots \;, \end{aligned} \tag{4.3}$$

where the zero modes of the gauge transformations $\sigma^{(\alpha_1\alpha_2)}, \sigma^{(\alpha_1\alpha_2\alpha_3)}, \dots$, are replaced by the redefined ghost fields $\hat{\sigma}^{(\alpha_1\alpha_2)}, \hat{\sigma}^{(\alpha_1\alpha_2\alpha_3)}, \dots$. Notice that, due to the redefinition of fields, the structure of zero modes of gauge transformations is not changed and the procedure outlined in the previous sections can be applied without any modifications. The only remaining problem is to show that, during the quantization procedure –which entails the introduction of non-minimal terms and canonical transformations– the gauge field V does not interact with the redefined ghost fields.

After the gauge fixing of the zero level, we noticed that a redefinition of fields and antifields is useful to cancel the couplings among the zero-level fields and those of the subsequent levels. Applying the canonical transformations in eqs. (3.7) and (3.11) to the interaction term in eq. (4.2), we find

$$
\begin{aligned}
&\mathrm{tr}\left(\hat{\bar{\sigma}}^{\dot{\alpha}}\bar{D}_{\dot{\alpha}}\left(e^{-V}-1\right)D_\alpha\hat{\sigma}^\alpha\right)\longrightarrow \\
&\mathrm{tr}\Big\{\left(\hat{\bar{\sigma}}^{\dot{\alpha}}\bar{D}_{\dot{\alpha}}\left(e^{-V}-1\right)D_\alpha\hat{\sigma}^\alpha\right)+\hat{\bar{b}}^{*\,\dot{\alpha}}_{\alpha}\,\frac{i\partial^{\alpha\dot{\tau}}}{\Box}\bar{D}_{\dot{\alpha}}\bar{D}_{\dot{\tau}}\left(e^{-V}-1\right)D_\beta\hat{\sigma}^\beta+ \qquad (4.4)\\
&-\hat{\bar{\sigma}}^{\dot{\beta}}\bar{D}_{\dot{\beta}}\left(e^{-V}-1\right)\frac{i\partial^{\tau\dot{\alpha}}}{\Box}D_\tau D_\alpha\hat{b}^{*,\alpha}_{\dot{\alpha}}+\hat{\bar{b}}^{*\,\dot{\beta}}_{\beta}\left[\frac{\partial^{\beta\dot{\rho}}}{\Box}\bar{D}_{\dot{\beta}}\bar{D}_{\dot{\rho}}\left(e^{-V}-1\right)\frac{\partial^{\rho\dot{\alpha}}}{\Box}D_\rho D_\alpha\right]\hat{b}^{*,\alpha}_{\dot{\alpha}}\Big\}\,,
\end{aligned}
$$

where the interaction with the gauge field and the antifield $\hat{b}^*$ emerges. This is harmless, if there are no additional gauge symmetries to be fixed. In fact, we have to fix the first level of ghost by means of the gauge fixing fermion

$$
\begin{aligned}
\Psi_2 \;=\;& \left(\hat{\bar{d}}^{\,(\dot{\alpha}\dot{\beta})}_{\alpha}\bar{D}_{\dot{\beta}}\hat{b}^\alpha_{\dot{\alpha}}+\hat{\bar{\nu}}\,i\,\partial^{\dot{\alpha}}_{\alpha}\,\hat{b}^\alpha_{\dot{\alpha}}+\hat{\bar{b}}^{\dot{\alpha}}_{\alpha\beta}\bar{D}_{\dot{\alpha}}\,\hat{\sigma}^{(\alpha\beta)}+\frac{1}{2}\hat{\bar{\mu}}\,C_{[\alpha\beta]}\,\hat{\sigma}^{[\alpha\beta]}+\mathrm{h.c.}\right)\\
+\;& \left(\hat{\bar{d}}^{\,(\dot{\alpha}\dot{\beta})}_{\alpha}\,\hat{b}^\alpha_{(\dot{\alpha}\dot{\beta})}+\hat{\bar{\nu}}\,\hat{m}+\hat{\bar{b}}^{\dot{\alpha}}_{\alpha\beta}\,\hat{\delta}^{(\alpha\beta)}_{\dot{\alpha}}+\hat{\bar{\mu}}\,\hat{n}+\mathrm{h.c.}\right)\,, \qquad (4.5)
\end{aligned}
$$

This shifts the field $\hat{b}^*$ with $\hat{b}^{*,\alpha}_{\dot{\alpha}}\longrightarrow\hat{b}^{*,\alpha}_{\dot{\alpha}}+D_\beta\hat{d}^{(\alpha\beta)}_{\dot{\alpha}}-i\,\partial^\alpha_{\dot{\alpha}}\hat{\nu}$ and, as a consequence of the D-algebra (cf. [4, 1]), it generates the new interaction terms

$$
\left\{\hat{\bar{\nu}}\left[(1-K_0)\bar{D}^2\left(e^{-V}-1\right)\right]D_\beta\hat{\sigma}^\beta+\mathrm{h.c.}+\hat{\bar{\nu}}\left[(1-K_0)\bar{D}^2\left(e^{-V}-1\right)D^2\right]\hat{\nu}\right\}\,. \qquad (4.6)
$$

These new terms introduce couplings among the gauge field V, the 0^{th}-level fields and the 2^{th}-level ghost fields $\hat{\nu}$. This can couple the "classical" fields with the complete tower of ghost fields, and thus it would destroy also the quantization procedure discussed in [1].

Pursuing the quantization procedure along the lines of [1], one has to diagonalize the 1^{st} ghost fields (see the table (1)) in order to eliminate the coupling among antifields and fields of different levels. It is easy, even though quite long, to show that, as a consequence of this diagonalization, the field ν undergoes the following redefinition

$$
\hat{\nu}\longrightarrow\hat{\nu}-i\frac{1}{\Box}\partial^{\dot{\alpha}}_{\alpha}\,\hat{b}^{*,\alpha}_{\dot{\alpha}}+i\frac{1}{\Box}D_\beta\,\partial^{\dot{\alpha}}_{\alpha}\,\hat{\beta}^{*,(\alpha\beta)}_{\dot{\alpha}}\,, \qquad (4.7)
$$

and its hermitian conjugate transforms accordingly. The first term of the redefinition of ν provides the contribution which, summed to corresponding terms of Eq. (4.4), cancels the coupling among b^* and the gauge fields up to zero modes. The second term does not introduce any further coupling as a consequence of D-algebra. Notice that this already truncates the coupling between the 0^{th}-level fields and the ghost tower.

Quantizing the 3rd level, one has to define an invertible kinetic term for the $\hat{\nu}$ field. Following [1], one has to introduce proper Nielsen-Kallosh ghost fields $\hat{\rho}_\alpha$ and their antifields $\hat{\rho}^*_{\dot{\alpha}}$. The canonical transformation that fixes the gauge of ν and diagonalizes the fields (eliminating the coupling between $\hat{\nu}$ and the antifields $\hat{\rho}^*_{\dot{\alpha}}$) amounts to a further redefinition of the $\hat{\nu}$ fields of the form

$$\hat{\nu} \longrightarrow \hat{\nu} + D_\rho \left(H^{\rho\dot{\beta}} \hat{\rho}^*_{\dot{\beta}} \right) , \tag{4.8}$$

where $H^{\rho\dot{\beta}}$ is an integral-differential operator whose expression is not relevant for our purposes. Again, this shift does not introduce any new coupling with the gauge fields. From this level on, no further redefinition or diagonalization can produce interaction terms with the gauge fields and the quantization procedure of [1] can be safely applied.

To conclude, we have proved that the quantization procedure of the complex linear superfield coupled to a gauge field can be performed along the lines discussed in the previous sections. Here, we stress the fact that, in order to respect the structure of levels and to avoid coupling among different types of ghost fields, an initial redefinition of the gauge field should be performed. The redefined fields show an infinitely reducible gauge symmetry of the same form as the non-gauged model. This resembles the main idea presented by P. Townsend in [10].

5 Conclusions

In the present paper, we reviewed the quantization procedure for theories with infinitely many ghost fields. In particular, we considered the complex linear superfield and a simple chiral (toy) model as examples. The quantization procedure is based on the conventional Batalin-Vilkovisky formalism, but suitable non-local canonical transformation are used in order to decouple fields of different ghost level. The computation of the antibracket cohomology is performed and it is shown that the technique provides the correct physical degrees of freedom. In addition, we considered the complex linear superfield model coupled to gauge fields, and we showed that also this model can be quantized along the same lines of the ungauged one.

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A Conventions

The superspace conventions we use are the same as in [1]; the spinor indices are raised with the charge conjugation matrix $C^{\alpha\beta}$. Some useful relations are:

$$
\begin{aligned}
D_\alpha \bar{D}_{\dot{\alpha}} + \bar{D}_{\dot{\alpha}} D_\alpha &= i\partial_{\alpha\dot{\alpha}} \\
D^2 &= \frac{1}{2} D^\alpha D_\alpha \\
\Box &= \frac{1}{2}\partial^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} = D^2\bar{D}^2 + \bar{D}^2 D^2 - D^\alpha \bar{D}^2 D_\alpha \\
D_\alpha D_\beta &= C_{\beta\alpha} D^2
\end{aligned} \tag{A.1}
$$

The table (1) represents the structure of the classical and ghost fields of the model. It is understood that each table should be doubled to account for the complex conjugates of all the fields. Fields which occur together in the gauge fermion are connected by a diagonal arrow.

All the fields and antifields can be assigned an helicity. In the extended action only fields of different helicity are multiplied together: any term in the action or in the gauge fermion is of the form $L\ O\ R$, where O is a possible superspace operator or spacetime derivative. All fields with upper dotted indices and lower undotted indices are right-handed, and vice versa for the left-handed ones. Fields and antifields have opposite helicities, and so do fields and their complex conjugates.

A delicate point is how one treats the commutation or anticommutation of fields and the superspace coordinate θ^α and D_α which are fermionic. The linear superfield Σ is bosonic, and, therefore, σ^α is fermionic. Since antifields have opposite statistics to fields, σ^*_α is bosonic. Then, all σ-fields are fermionic. The gauge fermion is always fermionic. It follows that $b^\alpha_{\dot{\alpha}}$ is fermionic and therefore $\sigma^{[\alpha\beta]}$ is also fermionic, and its ghost λ is bosonic.

B BFM and SUSY

We recall some features of the BFM in the SUSY context and two different way to introduce the splitting background-quantum in the SUSY context. We start from the discussion of the BFM in supersymmetry provided in [4] and we compare with the splitting discussed in [10].

The vectorial representation of the gauge superfield is defined by the covariant derivatives

$$
\nabla^v_\alpha = e^{-W} D_\alpha e^{W}\,, \quad \bar{\nabla}^v_{\dot{\alpha}} = e^{\bar{W}} \bar{D}_{\bar{\alpha}} e^{-\bar{W}}\,, \quad \nabla^v_\mu = -i\sigma^{\alpha\dot{\alpha}}_\mu \left\{\nabla^v_\alpha, \bar{\nabla}^v_{\dot{\alpha}}\right\}\,. \tag{B.1}
$$

Table 1: Ghost and Antighosts up to third level.

$$
\begin{array}{lccccccccc}
\mathrm{F} & & & & \underset{(0,-1)}{\sigma^{\alpha}} & & & & & \\
 & & & \swarrow & & & & & & \\
\mathrm{F} & & & \underset{(-1,0)}{b^{\dot{\alpha}}_{\alpha}} & & & \underset{(1,-2)}{\sigma^{\alpha_1\alpha_2}} & & & \\
 & & \swarrow & & & \swarrow & & & & \\
\mathrm{F} & & \underset{(0,-1)}{d^{A_2}_{\dot{\alpha}}} & \overset{*}{\longleftrightarrow} & \underset{(-2,1)}{b^{\dot{\alpha}}_{A_2}} & & & \underset{(2,-3)}{\sigma^{A_2\alpha_3}} & & \\
\mathrm{B} & & \nu & \overset{*}{\longleftrightarrow} & \mu & & & \lambda & & \\
 & \swarrow & & & \swarrow & & \swarrow & & & \\
\mathrm{F} & \underset{(-1,0)}{e^{\dot{A}_2}_{A_2}} & & \underset{(1,-2)}{d^{A_3}_{\dot{\alpha}}, d} & \overset{*}{\longleftrightarrow} & \underset{(-3,2)}{b^{\dot{\alpha}}_{A_3}, b} & & & \underset{(3,-4)}{\sigma^{A_3\alpha_4}, \varsigma} & \\
\mathrm{B} & \rho^{\dot{\alpha}} & & \nu^{\alpha} & \overset{*}{\longleftrightarrow} & \mu_{\alpha} & & & \lambda^{\alpha} &
\end{array}
$$

These derivatives are covariant with respect to the supergauge transformations

$$e^{W'} = e^{i\bar{\Lambda}} e^{W} e^{-iK} \,, \quad \bar{K} = K \,, \quad \bar{\nabla}^{v}_{\dot{\alpha}} \Phi^{v} = 0 \,. \tag{B.2}$$

The last equation defines a covariantly chiral superfield Φ. Another useful representation is the chiral one, which is obtained from (B.1) by *sandwiching* the covariant derivatives of the vector representation between $e^{-\bar{W}}$ and $e^{\bar{W}}$:

$$\begin{aligned} &\nabla^{c}_{\alpha} = e^{-\bar{W}} \left(e^{-W} D_{\alpha} e^{W} \right) e^{\bar{W}} = e^{-V} D_{\alpha} e^{V} \,, \qquad \bar{\nabla}^{c}_{\dot{\alpha}} = \bar{D}_{\bar{\alpha}} \,, \\ &\Phi^{c} = e^{-\bar{W}} \Phi^{v} \,, \qquad e^{V} = e^{W} e^{\bar{W}} \,. \end{aligned} \tag{B.3}$$

The superfield V transforms under a supergauge transformation in the following way:

$$e^{V'} = e^{W'} e^{\bar{W}'} = e^{i\bar{\Lambda}} e^{W} e^{-iK} e^{iK} e^{\bar{W}} e^{-i\Lambda} = e^{i\bar{\Lambda}} e^{V} e^{-i\Lambda} \,. \tag{B.4}$$

The prepotential V and the complex linear superfields Σ's transform, under gauge transformations, as

$$\begin{array}{rclccc} e^{V'} &=& e^{i\bar{\Lambda}} e^{V} e^{-i\Lambda} & e^{-V'} = e^{i\Lambda} e^{-V} e^{-i\bar{\Lambda}} & \bar{D}_{\dot{\alpha}} \Lambda = 0 \quad , & D_{\alpha} \bar{\Lambda} = 0 \\ \Sigma' &=& e^{i\Lambda} \Sigma e^{-i\Lambda} & \bar{\Sigma}' = e^{i\bar{\Lambda}} \bar{\Sigma} e^{-i\bar{\Lambda}} & & \end{array} \tag{B.5}$$

The action in terms of the new fields is given by

$$S = \frac{1}{\beta^2} \int d^4x \; d^2\theta \; d^2\bar{\theta} \; tr \left[-\frac{1}{2} (e^{-V} D^{\alpha} e^{V}) \bar{D}^2 (e^{-V} D_{\alpha} e^{V}) - e^{-V} \bar{\Sigma} e^{V} \Sigma \right] \,. \tag{B.6}$$

Table 2: Lagrange multipliers up to third level.

$$
\begin{array}{cccccc}
\mathrm{F} & & & \underset{(0,-1)}{\beta_{\alpha}^{\dot{\alpha}}} & & \\
 & & & \swarrow & & \\
\mathrm{F} & & \underset{(1,-2)}{\delta_{\dot{\alpha}}^{A_2}} & & \underset{(-1,2)}{\beta_{A_2}^{\dot{\alpha}}} & \\
\mathrm{B} & & n & & m & \\
 & \swarrow & & \swarrow & & \\
\mathrm{F} & \underset{(0,-1)}{\eta_{A_2}^{\dot{A}_2}} & & \underset{(2,-3)}{\delta_{\dot{\alpha}}^{A_3}, d} & & \underset{(-2,3)}{\beta_{A_3}^{\dot{\alpha}}, b} \\
\mathrm{B} & r^{\dot{\alpha}} & & n^{\alpha} & & m_{\alpha}
\end{array}
$$

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